Cellular Automata-Based Generative Design of Pied-de-poule Patterns using Emergent Behavior: *Case Study of how Fashion Pieces can Help to Understand Modern Complexity*

Loe Feijs [1,*] and Marina Toeters [2]

[1] *Faculty of Industrial Design, Eindhoven University of Technology, The Netherlands*
[2] *by-wire.net • design & research in fashion technology, Utrecht, The Netherlands*

Complex behavior arises from a multitude of interacting agents and even if the rules of the agents seem of simple design, the behavior of a crowd of agents can be overwhelmingly complex. The three cornerstones of complexity theory are emergence, transition and resilience. We argue that the fashion system is an example of a complex adaptive system. We focus on a particular type of fashion pattern known as Pied-de-poule (houndstooth) and use it as an inspiration to design cellular automata that generate new patterns (motifs). Cellular automata are tools in complexity theory, the science which aims at understanding complexity. The newly generated patterns feature emergence, transition and resilience, although they are only based on the simplest automaton that generates the simplest Pied-de-poule. These patterns are woven and the resulting innovative fabrics are used to design and construct a mini-collection of contemporary fashion items. The fashion items are meant to be fashionable and illustrate a new and modern understanding of complexity. We claim that programming is a new craft which is essential for a range of emerging new aesthetic possibilities in design and for developing new product semantics. We describe how the coding process is integrated with the fashion design, with many iterations in the coding phase and multi-disciplinary cooperation in the overlapping weaving, design and construction phases.

Keywords – Complexity Theory, Parametric Design, Cellular Automata, Pied-de-poule, Houndstooth, Fashion System.

Relevance to Design Practice – Programming is a new craft which is essential for a range of emerging new aesthetic possibilities in design and for developing new product semantics. The designed garments encode the message that there is a new understanding of complexity which is relevant for many disciplines, including fashion and design itself.

Citation: Feijs, L., & Toeters, M. (2018). Cellular automata-based generative design of Pied-de-poule patterns using emergent behavior: Case study of how fashion pieces can help to understand modern complexity. *International Journal of Design, 12*(3), 127-144.

Background

In this paper we present results of our exploration to generate innovative fashion patterns (motifs) which express an understanding of complexity. Present-day research into complexity is a multidisciplinary area where mathematicians, biologists, physicists, epidemiologists and economists, amongst others, cooperate to get a grip on complexity. Complex behavior arises from a multitude of interacting agents and even if the rules of the agents are not very complex, the behavior of a crowd of agents can be overwhelmingly complex. The three cornerstones of complexity theory are: emergence, transition and resilience (Vermeer, 2014). The research area is considered both societally relevant and promising.

The societal relevance of complexity is related to the fact that human-made systems increasingly operate at a global scale. Epidemics, economic growth, pollution and biological diversity can no longer be considered local problems. Both the effect scale (global instead of local) and the time scale (seconds instead of weeks) are changed by the unprecedented growth of digital connectivity (telegraph, telephone, wireless, Internet, Internet of Things). Modelling and simulation are important ways of working in complexity research and so-called cellular automata are some of several options for modelling and simulation (alongside evolutionary algorithms, neural networks and network models). Cellular automata have been used for studying group behavior (Bin & Zhang, 2006), traffic jams (Castillo et al., 2016), pedestrian movement (Guan, Wang & Chen, 2016), drug dissolution modelling (Bezbradica et al., 2016), leader election problems (Banda, Crane & Ruskin, 2015), artificial life (Langton, 1986), and so on.

The goal of our exploration is to design a two-dimensional pattern which is potentially applicable in fashion and which contains references to a modern understanding of complexity. We shall weave the pattern on a Jacquard loom (which has a place in the history of computers).

Received June 30, 2017; **Accepted** October 14, 2018; **Published** December 31, 2018.

****Corresponding Author:*** L.M.G.Feijs@tue.nl

In order to strengthen the applicability and to keep a visible link to the existing fashion culture, we demand that the pattern has a recognizable connection to a well-known and expressive pattern, for which we choose Pied-de-poule. We shall define explicitly what constitutes a *Pied-de-poule like pattern* in the fourth section of this article.

There is a heavy emphasis on this particular weave throughout this article, so we should ask whether the approach will generalize to other woven patterns or even patterns outside the weaving tradition. We choose Pied-de-poule and take advantage of the fact that its mathematics is well-understood. This makes it possible to make a connection to complexity, not only at the level of visual effects, but also at the level of the underlying mathematics. We could choose other woven patterns such as herringbone, goose eye, or Prince of Wales, which are equally suitable as a starting reference and we would be able to establish similar connections to complexity, both at the visual and the mathematical level. The same techniques used in this article are applicable. There are other famous fashion patterns, unrelated to weaving, such as Paisley print, Celtic knot and floral patterns, for which it would be fascinating to explore such connections; but these would be new projects and we can only speculate about the visual and mathematical findings. In this work we focus on Pied-de-poule, leaving other patterns as options for future research.

Both in fashion and in industrial product design, meanings are expressed not by text, but by the form: the texture, materials, colours and the shape itself. The art and science of expressing meanings in such manner is called product semantics (Feijs & Meinel, 2005; Krippendorff, 1989). Peirce (Peirce, 1931) describes three ways in which (product) semantics work: iconic (product looks like...), symbolic (product is a learned code for...), and indexical (product looks like a trace of...) (Chandler, 2003). Eco distinguishes symbol use into two types: ratio-difficilis (first usage, introducing a new symbol) and ratio-facilis (using a symbol from a well-accepted code) (Eco, 1979). In the to-be-designed pattern of this project we envision an iconic meaning for both the Pied-de-poule character and for the reference to complexity theory. But modern complexity theory is not well-known among the general public so depending on the context, we will introduce it as a symbolic meaning of a ratio-difficilis kind first.

Loe Feijs (1954) received the M.Sc. degree in electrical engineering and the Ph.D. degree in computer science from the Eindhoven University of Technology. In the 1980s, he designed for video compression and telephony systems. He joined Philips Research to develop formal methods for software development. He became a part-time Professor of mathematics and computer science in 1994, the Scientific Director of the Eindhoven Embedded Systems Institute in 1998 and the Vice Dean of the Department of Industrial Design in 2000. He is currently a Professor of industrial design at TU/e. He has authored three books on formal methods & design and over 200 scientific papers. His current research interests include creative programming, design of wearable systems and biofeedback systems.

Marina Toeters (1982) is educated as a graphic and fashion designer and finished her Master of Art cum laude at MAHKU Utrecht by exploring the gap between designers and technicians in the world of fashion. She motivates collaboration for fashion innovation and is initiator of by-wire.net • design & research in fashion technology, working amongst others for Philips Research and European Space Agency (ESA). Toeters is a member of the research group Smart Functional Materials at Saxion University for applied science and teaches New Production Techniques for textile & garments. She is coach in Wearable Senses, Industrial Design faculty, at the Eindhoven University of Technology and lecturer Fashion Ecology & Technology at the University for Art and Design Utrecht.

The article is laid out as follows: after an introduction to fashion as a complex system and an overview of related work, we present Pied-de-poule and cellular automata. Next, we develop a family of cellular automata which have non-trivial emerging behavior patterns which at the same time resemble Pied-de-poule. Then, we construct a small collection of contemporary and attractive fashion items, using the new pattern, expressing the message of being fashionable and presenting a new and modern understanding of complexity.

Fashion as a Complex System

The fashion system itself is a complex adaptive system, as we shall argue now. It can be described as a dynamic system with various feedforward and feedback loops. A first feedforward model is the trickle-down effect that celebrities and high-class people adopt new fashion first, other people following them with some delay. Then, there are seasonal effects: during winter people need warm clothes and choose dark colors, for summer they need light-weight garments and choose brighter colors. There are also trends which are induced by changes outside of fashion because designers and consumers are for example influenced by the economy; the controversial hemline index goes back to 1926. The hemline index is discussed by amongst others Kim and Ahn (2015). Fashion designers code their ideas about society into their collections. Opinions on what is beauty, even beauty of the human body, are different per culture and subculture and change over time. New materials and technologies come into existence and are subsequently used for fashion.

Most of these models are based on feedforward mechanisms (Crane, 1999), but there is a growing number of feedback mechanisms. One of the oldest models is about snobs (observing others, avoiding similarity) and individuals trying to dress similar to others (bandwagon-effect). Then, there are the fashion firms copying each other's ideas. Miller, McIntyre and Mantrala (1993) give a formal description of several of the above mentioned feedback loops using matrices of weighting factors. Professional trend forecasting firms observe the fashion system, the economy, the art world and many other factors to predict the colors, fabrics and cuts several years ahead. Their intelligence is bought by fashion firms and other trend sensitive companies to inform their design decisions. Kosztowny (2015) gives a good overview of how trend forecasting firms work. The end-users are creative as well, creating items by DIY, unconventional re-use, color combinations, haircuts, tattoos, and so on. They are observed by the trend forecasters, but also by magazines and bloggers, and circulated on social media and thus fed back into the system. Fast fashion brands (H&M, Zara) shorten the production cycles and adapt within weeks to market responses. The behavior of all this is oscillating and hard to predict (if commercial parties could predict well, there would be no competitive advantage left). The number of active agents is growing quickly and the fashion system itself is therefore an example of a complex adaptive system.

Several authors confirm this view: Law, Zhang and Leung (2004) argue that fashion consumption is chaotic. Frederiksson (2008) describes the various roles such as anti-innovators, conservatives, trend creators, trendsetters, mediocre trend followers and mainstreamers (referring to swarm behavior and the butterfly effect of chaos theory). Laurell (2016) describes the complexity as a number of fashion spheres where users build networks and negotiate meaning. Edelkoort (2015) presents a remarkable and critical perspective on the fashion system. The critique is not about one agent such as a fashion brand, fashion school, or factory. The commentary is that the entire system, with its interlocked dependencies, has evolved in a very unfortunate direction.

Occasionally fashion designers use their medium par excellence, the garments presented in the fashion show, to express their interest or concern about a complexity-related societal phenomenon. Hussein Chalayan, for example in his Fall/Winter 2000 show, addressed the themes of migration and mobility, see Quinn (2000). Viktor and Rolf did not directly address complexity and stress, but instead showed their opposites, simplicity and serenity, in their 2013 show called Instant Zen garden, as described by Feiereisen (2013).

The complex adaptive systems (CAS) community considers simulation as a powerful tool for gaining understanding. Simulation is useful for quantitative prediction, typically for logistic challenges in (fashion) production chains. For example Cagliano, DeMarco, Rafele and Volpe (2011) obtain performance improvements for centralized warehousing using system dynamics simulation.

Another type of simulation is Troy Nachtigall's (2017) *Life of Fashion Trends*, which is descriptive rather than quantitative. Besides building a fashion trend simulator based on Conway's *Life*, Nachtigall wrote a realistic blog that describes the events in a simulation run, observing emergent behavior using terms such as the movement of trends, the hotspot and notspot.

Related Work

There is a tradition of designing innovative garments which announce technological possibilities and pave the way for commercial applications. Iris Van Herpen did this with 3D printing in fashion, see Kuhn & Minuzzi (2015). Hussein Chalayan did this with embedded actuators in garments, see Quinn (2000). Pauline Van Dongen did it with wearable displays (flip-dot dress), see Van Kessel (2013). The general pattern of these innovations is as follows: there is a new technology for which the innovative garment proves the potential under new functional, semantic and aesthetic demands (outside the context of the lab, where the technology is tested amidst a mess of wires and instruments). Many examples could be found on the exhibitions Pretty Smart Textiles (see http://prettysmarttextiles.com/exhibition2012belgium/) and Coded Cloth (Rackham, 2009).

There are examples, though not many, where mathematics is seen as a technology, such as the works of Tenthof Van Noorden et al. (2014) and joint work by Gabriela Ligenza (2015) and De Comité (2014). Pied-de-poule was used as a starting point to make sophisticated patterns, adding mathematical principles (recursion, turtle-graphics, Lindenmayer systems and sphere packing) in (Feijs & Toeters, 2013, 2015b, 2016). The central theme of the added principles was fractals and digital production methods were deployed, notably laser cutting computer controlled embroidery. Such added principles give rise to new aesthetics and at the same time they act as references to the classic Pied-de-poule and present a first glimpse of new ideas on complexity. More precisely, fractals feature a special symmetry: scale-invariance, which appears more modern than the old school symmetries such as translation, rotation, mirroring and glide mirroring. Doug Blumeyer presents a large collection of Pied-de-poule variations on his website cmloegcmluin (see https://cmloegcmluin.wordpress.com/2018/08/13/houndstooth-taxonomy/) including a generator called houndstoothcraft to combine various patterns and a number of innovative fractals with names such as thousoondth and holestooth.

Some garments were created where complexity itself is proposed as a technology, but not many. First there are straightforward commercial print applications of the well-known Mandelbrot set. Closer to our work is Fabienne Serriere's kickstarter *KnitYak*, producing custom mathematical knit scarves. Working with mathematicians Elisabetta Matsumoto and Henry Segerman, KnitYak produced beautiful Möbius cellular automata scarves (Matsumoto, Segerman & Serriere, 2018).

Nervous System (http://n-e-r-v-o-u-s.com) draws inspiration from natural phenomena, creating computer simulations to generate designs and use digital fabrication to realize unique jewelry products. In McBurney (2009) an example is given of a simple weaving pattern generated by a cellular automaton, but neither a garment nor a discussion of complexity is presented. In Holden and Holden (2016) examples are given of fiber art in braids, cables and weaves with cellular automata.

The science of complexity is growing fast, witnessed by new journals such as *Complexity* (Wiley-Interscience), *Computational complexity* (Springer), *Ecological complexity* (Elsevier), *Journal of complexity* (Elsevier), *Journal of systems science and complexity* (Springer) and *Complex systems* (Complex Systems Publications).

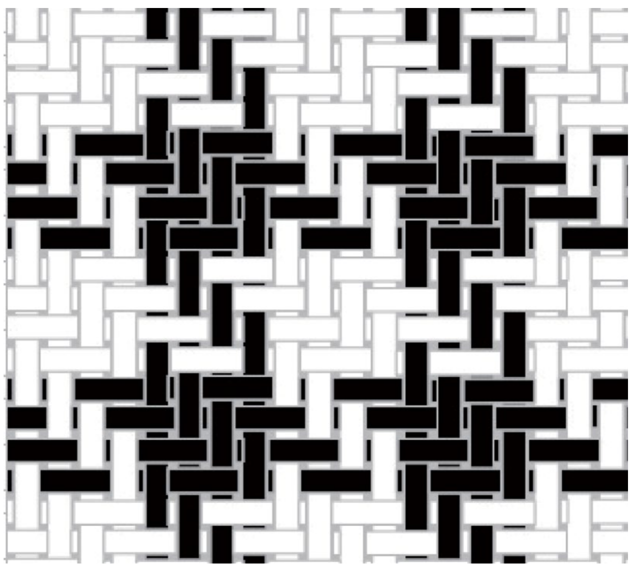

Figure 1. Classical Pied-de-poule pattern emerging from twill weaving.

What is Pied-de-poule?

In Franklin (2012), we find: "A twill weave in which two colours of yarn are used to create a broken checked pattern or a pattern of abstract, four pointed-shapes" (p. 446). More precisely, Pied-de-poule, also called houndstooth, is the textile pattern produced by weaving black and white yarns in alternating blocks, both in the warp and the weft, using a balanced twill binding such that the block size is twice the float length, thus appearing as shown in Figure 1 (which indicates how the weaving works. Source: Wikimedia) and Figure 2 (which shows that certain variations exist). We shall explain the technical terms such as warp, weft, twill, etc. below. Other contrasting colors can be used, but usually, the colors are black and white. The pattern's visual appearance is very strong and the characteristic broken checks are easy to recognize. The pattern cannot be overlooked. Although Pied-de-poule patterns can be printed or knitted, its origins are indisputably in weaving (Feijs, 2012; Gennert Jakobsson, 2018; Wilson, 2012).

Pied-de-poule has a long history and the oldest find is the Gerum cloak (Sweden), which has been radiocarbon dated to 360,100 BC, the pre-Roman iron age (Frei, 2009). Pied-de-poule was introduced in fashion by the Prince of Wales (Edward the VII) in the 1930s and in haute couture by Dior in the 1950s. Ever since, until today it is frequently used in haute couture, prêt-à-porter and mass-produced fashion. Although the classic pattern is old, the same idea is recycled over and over again in different contexts, different cuts and different combinations. Pied-de-poule is very much alive as shown in Figure 3, featuring celebrity Queen Máxima of The Netherlands (image courtesy EM Press/Van Emst).

Weaving a classical Pied-de-poule is done on a machine called a loom (Gandhi, 2012) in the following way: there is one set of yarns in longitudinal direction which is called the warp, and one set of yarns in the orthogonal direction which are maneuvered or shot one by one through the warp. The latter set of yarns is called the weft. When a so-called plain binding is used, the weft yarn goes over one warp yarn, then under the next warp yarn, then over again, then under, and so on. In another type of binding, the weft yarn goes two-over, two-under, and so on. This is shown in Figure 1. In general, such a weaving with $N1$ over, $N2$ under for $N1 > 1$ or $N2 > 1$ is called a twill binding if the over/under pattern shifts by one for each consecutive weft. If $N1 = N2 = N$ the twill is said to be balanced and the number N is called the float length. Now, alternatingly use black and white warp yarns, say k black yarns, k white yarns, and so on; in the same way use alternatingly black and white weft yarns, say k black yarns, k white yarns, etc. again. When such warp and weft color scheme is deployed in combination with plain binding, the classical block patterns typically used for towels, carpenter shirts, and chef trousers appear. When such warp and weft color scheme is deployed in combination with twill binding, a more complicated pattern arises. In particular, taking $k = 2N$, we get Pied-de-poule (houndstooth). There is a family of Pied-de-poule patterns (Feijs, 2012), one pattern for each integer $N > 0$. The variation displayed in Figure 2 can now be explained very precisely: these are the Pied-de-poules for $N = 1$, 2 and 3.

The Pied-de-poules in Figure 2 have been generated by the simple Mathematica program shown in Figure 4. The case $N = 1$ is ambiguous in the sense that it is both a block pattern and a Pied-de-poule pattern (Feijs, 2012). The case $N = 2$ is the same as in Figure 1 (but the over/under effect is flattened). Beyond $N = 1$, 2, 3, 4, …, there is a pattern which arises as a limit case $N \rightarrow \infty$, although this cannot be woven; it can be printed or laser-cut, however. The mathematics of Pied-de-poule was previously analyzed in Feijs (2012) and Ahmed (2014).

What is a Cellular Automaton?

A cellular automaton is a model of a system of cell objects with the following characteristics (Shiffman, Fry & Marsh, 2012): the cells live on a grid; each cell has a state; the number of state possibilities is finite; each cell has an environment (neighborhood) which is a list of adjacent cells and the new state value is obtained by a rule from the previous environment states. The simplest cellular automata are one-dimensional, but two, three and higher dimensional cellular automata can be defined as well. The following three principles apply to cellular automata (Schiff, 2011): homogeneity: all cell states are updated by the same set of rules; parallelism: all cell states are updated simultaneously; locality: the rules are local in nature.

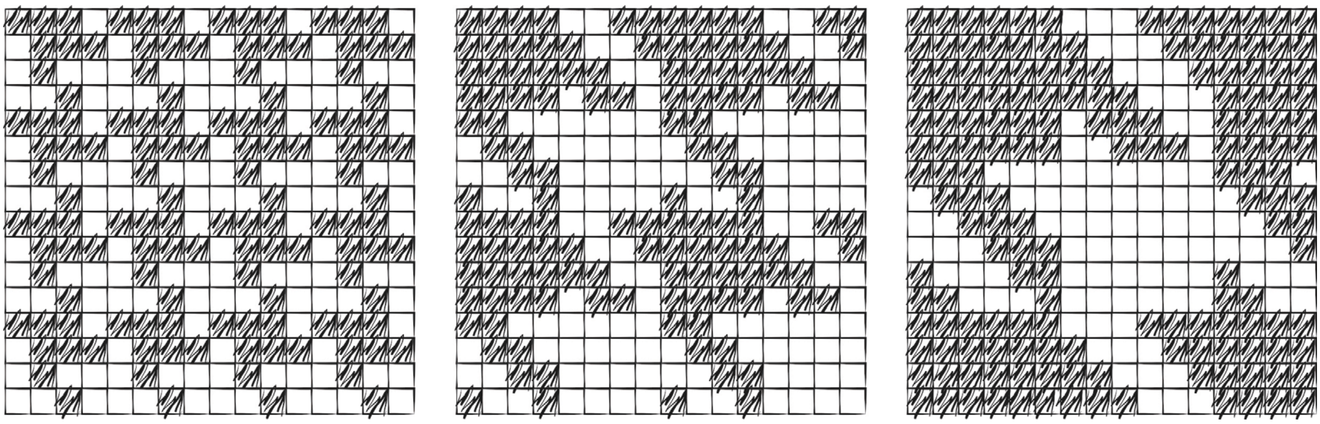

Figure 2. Pied-de-poules for *N* = 1, *N* = 2 and *N* = 3.

Figure 3. Queen Máxima in Pied-de-poule coat (photo © and courtesy EM Press/Van Emst).

```
n := 1; (*pied-de-poule type N*)
SIZE := 16; (*weaving area*)
GRID :=  8; (*display only*)
Bit[b_] := If[b, 1, 0];

pdp = Table[If[Mod[2 × 3 × 4 × 5 + i - j, 2 n] < n,
               Bit[Mod[i, 4 n] < 2 n],
               Bit[Mod[j, 4 n] < 2 n]],
                      {j, 0, SIZE - 1}, {i, 0, SIZE - 1}];

ArrayPlot[pdp, ImageSize → SIZE × GRID, Mesh → True]
```

Figure 4. Mathematica program to produce a Pied-de-poule pattern for *N* = 1 (which can be adapted to *N* = 2 and *N* = 3).

Most of the dynamical features of cellular automata can be found in the study of the one-dimensional case (Schiff, 2011). As Wolfram (2002) puts it: "sometimes essential properties can already be observed in 1D. But cellular automata—and especially 1D ones—make the phenomena particularly clear" (p. 880). Therefore, in this project we focus on the design of a one-dimensional cellular automaton. In the one-dimensional case, an environment is defined by its radius r such that $r = 1$ means that each environment consists of 3 cells. In general for $r \geq 1$ each environment has $2r + 1$ adjacent cells. At each point in time, $t = 1, 2, 3\ldots$ each cell has a value which we call a state. The state can assume a set Q of distinct values (for example $Q = \{0, 1\}$ is a state space with two values, which we conveniently identify with colors, putting white = 0 and black = 1).

As an example of an automaton, the update rule is given visually in Figure 5. It could be expressed as a set of maplets $\{1, 1, 1\}\to 0$, $\{1, 1, 0\}\to 0$, $\{1, 0, 1\}\to 0$, $\{1, 0, 0\}\to 1$, $\{0, 1, 1\}\to 1$, $\{0, 1, 0\}\to 1$, $\{0, 0, 1\}\to 1$ and $\{0, 0, 0\}\to 0$. We use the convention of (Wolfram, 1999) that { and } denote tuples (lists). The evolution of any one-dimensional cellular automaton can be illustrated by starting with the initial state (generation one, $t = 1$) in the first (top) row, the next generation on the second row, and so on (Weisstein, 2002).

Towards Simulated Weaving

Before returning to Pied-de-poule, we explore how to design cellular automata whose output is like a given woven pattern. We give a few rules of thumb, starting with the simplest examples, that tell how to design an automaton to realize a user-specified design pattern. The rules of thumb are a sufficient starting point for Pied-de-poule. However, the general task of designing cellular automata for arbitrary specified patterns is a huge territory that is mostly uncharted. A cellular automaton's behavior is hard to predict, although we can run simulations. Some serendipity is helpful, as the world of cellular automata is full of surprises.

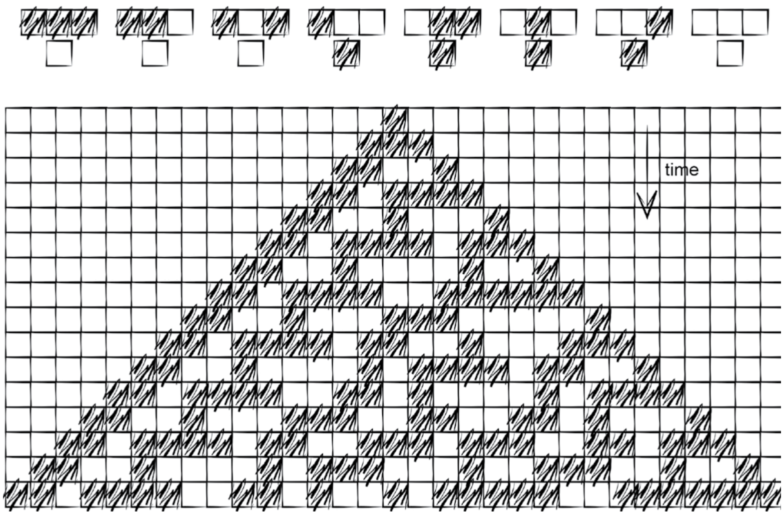

Figure 5. Example of a cellular automaton (Wolfram's rule 30).

By definition, in a plain binding (plain weave), the weft (vertical) yarn goes over one warp (horizontal) yarn, then under the next warp yarn, then over again, then under, and so on. Consider only the specific case of $k = 1$, i.e., when both the warp yarns and weft yarns are alternatingly black and white for every 1 yarn. Weaving thus we get horizontal or vertical stripes.

Horizontal stripes are easy to simulate by a 1D cellular automaton: take any rule which maps:

> {1,1,1}→0 and {0, 0, 0}→1 and feed it an initial row of all 1s (Wolfram's automata 1,3,5,7, …127 all can do this). Vertical stripes are easy too: deploy {1, 0, 1}→0, {0, 1, 0}→1 on an initial row of alternating 1s and 0s.

Staying with plain binding, we choose a more interesting pattern, small blocks, which is obtained by having an all-black warp and an all-white weft (no k needed). This produces a block pattern that has blocks of size 1 × 1. Which cellular automaton will produce this block pattern and which initial first row do we need? To avoid problems near the left and right edges, we reconnect the beginning and the end of each row, so our working area is cylindric. To make the automaton's task easier, we begin with a row of alternating zeros and ones. Looking for combinations of length three in the first row, we find two cases to be handled: {0, 1, 0} and {1, 0, 1}. Reading the desired result from the second row, we add two maplets:

> {0, 1, 0}→0 and {1, 0, 1}→1. Checking the second row, we find no new combinations. We then add default maplets (mapping anything else to zero). Thus we constructed an automaton which we show in action in Figure 6 (left).

This automaton can sustain the pattern from a properly filled initial row (we designed the maplets to do precisely that, nothing more). But this automaton does not generate the pattern; if we feed it a row with a single 1, nothing will appear.

So our next challenge, the first really interesting challenge, is to improve our automaton so it will generate the small-blocks pattern from a single seed. First, we check what needs to be done to get two blocks in the second row. We add two more maplets, one to let the pattern expand leftward into empty space and a second to let the pattern expand to the right: {0, 0, 1}→1 and {1, 0, 0}→1. Next, we check the third and fourth rows, but as no new combinations appear, we are done. Again, anything else is mapped to zero. If we launch the improved automaton with an initial row having a single 1, the automaton generates the block pattern, expanding with the *speed of light*, one cell per step. This expanding pattern is shown in Figure 6 (center).

Now, the question is what happens if we feed it an initial row with a few remote seeds. The outcome depends on the relative position of the seeds. If their distance is even, their generated outputs will merge nicely. Otherwise, they self-organize in vertical zones, as in Figure 6 (right).

Next, we undertake yet another challenge: say we have a K by K square grid of cells, for example, 12 by 12. Suppose we want the two diagonals to be black and all the other cells white. Can we design a 1D automaton and an initial first row to realize this goal? If we work with two colors, the answer is *no*, because, in a finite environment, there is no way to tell for a given black cell (state 1) to which of the two branches of the pattern it belongs. For fixed K = 12, a radius of 6 works, yet fails for larger K.

We can also solve this challenge, by working with more than two states (colors). We can differentiate the black into three different *shades of black*, say: dark-gray, dark-red, and dark-green. For coding we choose numbers: white = 0, dark-red = 1, dark-green = 2, and dark-grey = 3. If we insist that the shades of black really appear black, we can take very dark colors. The transition from the first to the second row is done by {0, 0, 3}→1 and {3, 0, 0}→2 (the dark-gray cell produces two branches). The dark-red and dark-green branches travel leftward and rightward by {0, 0, 1}→1 and {2, 0, 0}→2, respectively. Finally, {2, 0, 1}→3 takes care for merging the two branches (needed after K/2 steps).

If we allow ourselves to see all *shades of black* as black, the automaton solves the challenge. If we launch it with different initial rows, we get all kinds of different results, two of which are presented in Figure 7. Note that Figure 7 (left) has the same initial row as Figure 6 (center), except for a horizontal shift (and using

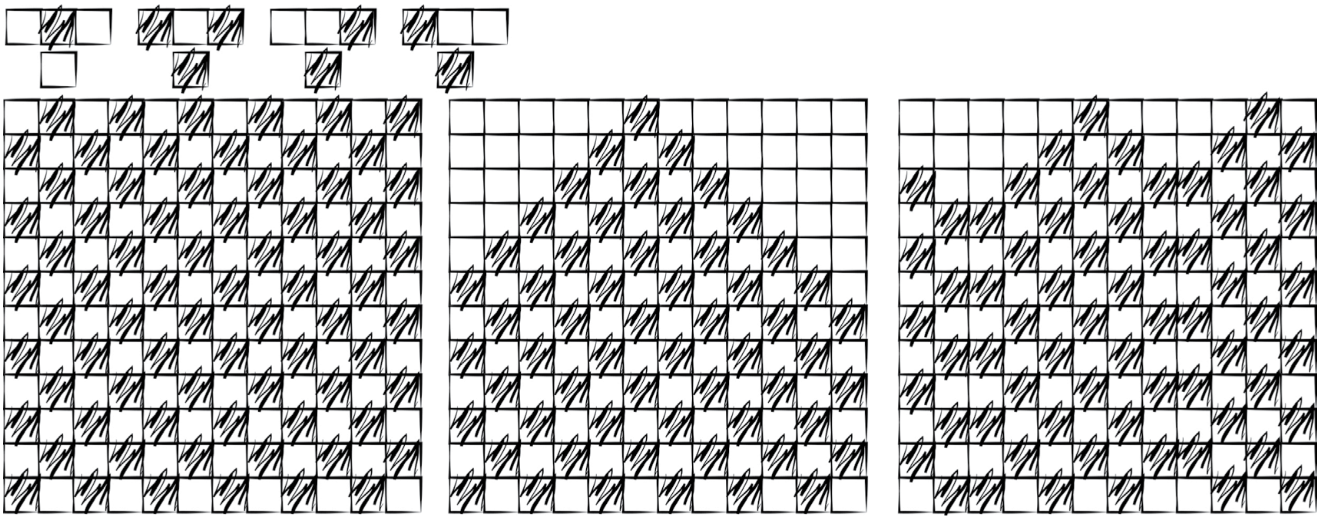

Figure 6. Automaton for generating a simple block pattern with selected initial seeds.

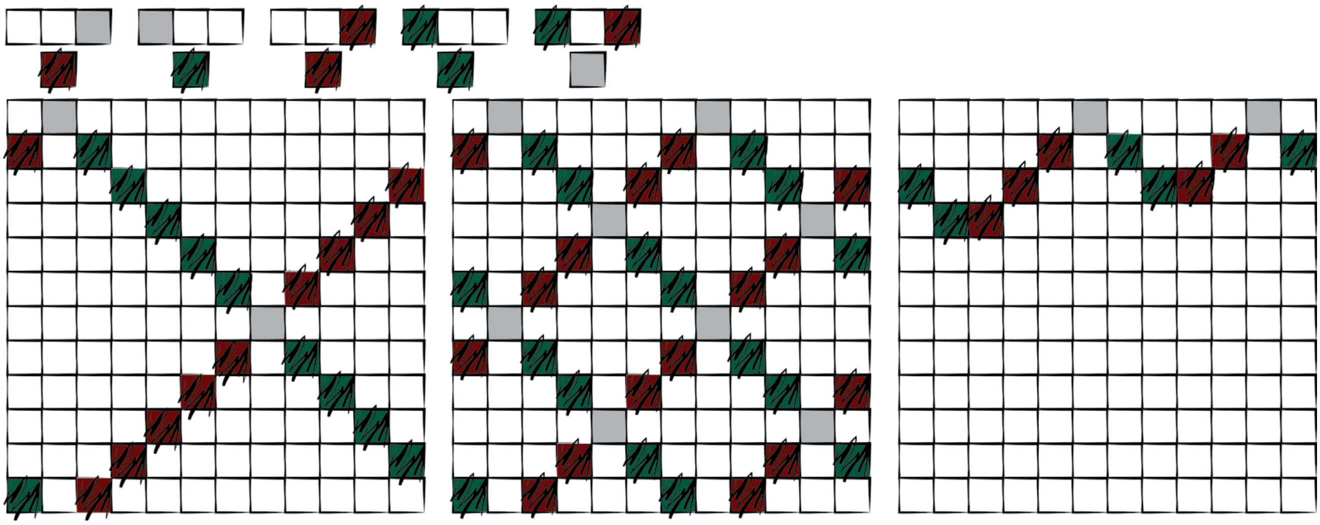

Figure 7. Automaton for making diagonal patterns with selected initial seeds.

dark-gray instead of black). Also note that Figure 7 (right) has the same initial row as Figure 6 (right). The two branches cancel out, but if we do not want that, we could add extra maplets to get the automaton going again.

Next, let us move to case $k = 2$ (two over, two under, and so on) and see how more powerful is it vs. $k = 1$. We could for example try to make bigger blocks of size 2 by 2, which can be woven by using a white weft and a black warp as follows: two over, two under, etc. for the first and second weft, followed by two under, two over, etc. for the third and fourth weft. This example is like a doubled plain binding (also known as basket weave). Trying to simulate this with an automaton we encounter a problem: the same patterns appear in consecutive rows, yet demanding different follow-ups. Therefore, colors are indispensable, only enlarging the radius does not help.

We illustrate this usage of color in Figure 8. The task is to make blocks of size 2 by 2. We use two different *shades of black*: dark-red = 1, and dark-green = 2, next to two "shades of white" (pinky and greeny, coded as −1 and −2). Moreover, there is still empty space = 0. For constructing the automaton we use the same two steps. The first step is to add sufficient maplets to sustain the pattern (maplets $\{-1,-1,1\}\to-2$, $\{1,-1,-1\}\to-2$, $\{-1,1,1\}\to2$, $\{1,1,-1\}\to2$, $\{-2,-2,2\}\to1$, $\{2,-2,-2\}\to1$, $\{2,2,-2\}\to-1$, and $\{-2,2,2\}\to-1$). The colors allow the automaton to perform a kind of line counting: red-like indicates an odd line, green-like indicates even. The second step is to add maplets so that, from the initial seed, patterns expand properly leftward and rightward. For example $\{0,0,1\}\to2$, $\{0,1,0\}\to2$ and $\{1,0,0\}\to-2$ will bring us from the first row with a single 1 to the next row. Carrying on like that we find the desired automaton.

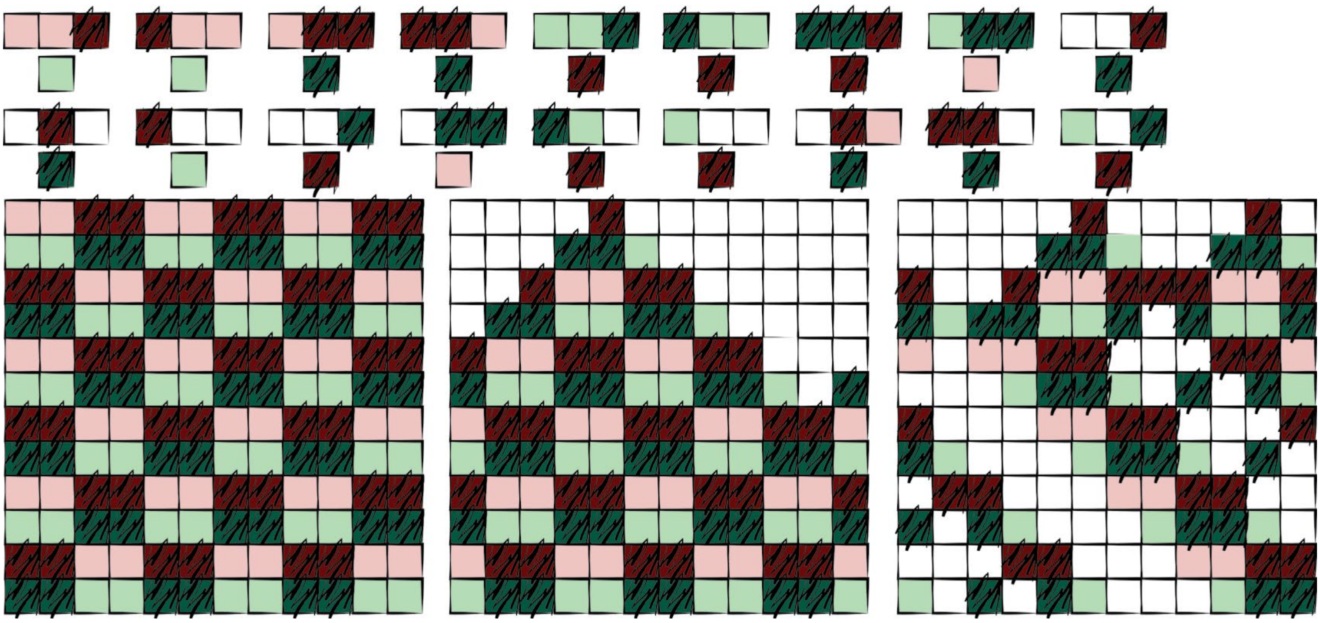

Figure 8. Automaton for making large blocks with selected initial seeds .

In summary, these are our rules of thumb: (1) from the given pattern, read the maplets needed to sustain the pattern, (2) add more maplets so the automaton will grow the pattern from a single seed (3) if these steps turn out impossible, enlarge either the environment or the set of states (more colors). Step (4) is to look for unexpected emergent behavior for other initial rows, such as cancelling-out, merging or self-organisation. Step (5) is to add additional maplets for extra effects. In the next section, these rules of thumb are applied to Pied-de-poule.

Designing an Automaton for Pied-de-poule

The obvious initial idea was to design a two-dimensional automaton such that at each point in time there is a two-dimensional grid, which resembles a Pied-de-poule pattern in some areas and which evolves to locally resemble such pattern every now and then. We found rules which would sustain a given Pied-de-poule pattern and we managed to add rules with a limited error-correction capability. But growing fresh Pied-de-poule patterns from random seeds was harder. Therefore, we switched to one-dimensional automata and found ways to obtain Pied-de-poule patterns. Then, taking notice of Schiff's remark that *most of the dynamical features of cellular automata can be found in the study of the one-dimensional case* and similar claims by Wolfram, we decided that it was perfectly okay to work in one dimension and we stuck to that for most of the exploration reported in the present paper. We need a rule, a recipe telling how a cell is updated as a function of its environment, so for environments of three cells:

> ($r = 1$ and $Q = \{0, 1\}$), the rule should describe 8 cases. One such case could be $\{0, 0, 0\} \rightarrow 0$, (as before, we call that a maplet). In general, a complete rule has q2r+1 maplets where q is the number states in Q and r is the radius that defines the environment.

Figure 9 shows that no rule can perform well in making Pied-de-poules at $t = 1$ and $t = 2$ since at $t = 1$ there is a need for $\{1, 0, 1\} \rightarrow 1$ whereas at $t = 2$ it should be $\{1, 0, 1\} \rightarrow 0$. Similar situations arise for $\{0, 0, 0\}$ for example. State 0 is plotted white, 1 as black. In fact, the problem persists if the environments are chosen larger as $t = 1$ and $t = 2$ are the same row of states, except for a horizontal shift (and similarly for $t = 3$ and $t = 4$). The problem persists for any $r \geq 1$. The proposed approach is to use five states ($q = 5$). There is one quiescent state, serving as the blank space where no Pied-de-poule (or anything else) has developed yet. Its color is pure white. Moreover, two extra kinds (shades) of white and two kinds of black are introduced in order to distinguish consecutive rows inside the Pied-de-poule (preventing the problem of Figure 9). White and black are the colors par-excellence for Pied-de-poule in fashion. Therefore, we adopt two light colors (called pinky and greeny) and two dark colors (dark-red and dark-green). The formal forgetful mapping F(dark-red) = F(dark-green) = black and F(pinky) = F(greeny) = white gives the classic black-and-white Pied-de-poule. We say pinky and dark-red are red-like, greeny and dark-green are green-like. The coding is: quiescent = 0, pinky = −1, greeny = −2, dark-red = 1, dark-green = 2. In other words, negative values are kinds of white, strictly positive values are kinds of black. The plan is to design an automaton such that it can evolve into (regions of) Pied-de-poule, in which red-like and green-like rows alternate.

Figure 10 shows how a rule of 9 maplets could produce two more rows from an initial grid with a single dark-red cell. Formally we let $r = 1$, $Q = \{-2, -1, 0, 1, 2\}$ (as a set) and then the 9 maplets are $\{0, 0, 0\} \rightarrow 0$, $\{0, 0, 1\} \rightarrow 2$, $\{0, 1, 0\} \rightarrow -2$, $\{1, 0, 0\} \rightarrow 2$, $\{0, 0, 2\} \rightarrow -1$, $\{0, 2, -2\} \rightarrow -1$, $\{2, -2, 2\} \rightarrow -1$, $\{-2, 2, 0\} \rightarrow 1$ and $\{2, 0, 0\} \rightarrow -1$. Continuing the development of Figure 10 we find that it takes 35 maplets to complete the emerging triangle (which begins with a single cell in state 1, dark-red). Of these maplets, 19 take care of growth at the edge of the blank areas (for example $\{0, 0, 1\} \rightarrow 2$) and 16 other maplets sustain the development inside the Pied-de-poule area (for example $\{2, -2, 2\} \rightarrow -1$, in which no 0 occurs). As a complete rule must have q2r+1 = 53 = 125 maplets (q being the number states in Q) we have considerable freedom what to do with the remaining 125 − 35 = 90 maplets. As a default rule we map everything else to the quiescent state $\{\_, \_, \_\} \rightarrow 0$, which is shorthand for the 90 maplets (everything else→0). This formally defines our automaton. More precisely, we have one automaton for each grid size (initial row length) L. The grid is organized circularly so that the first and the last cells are neighbors. We often work with grid lengths L which are multiples of 4 (the largest grid size we used in our computations so far is 2048).

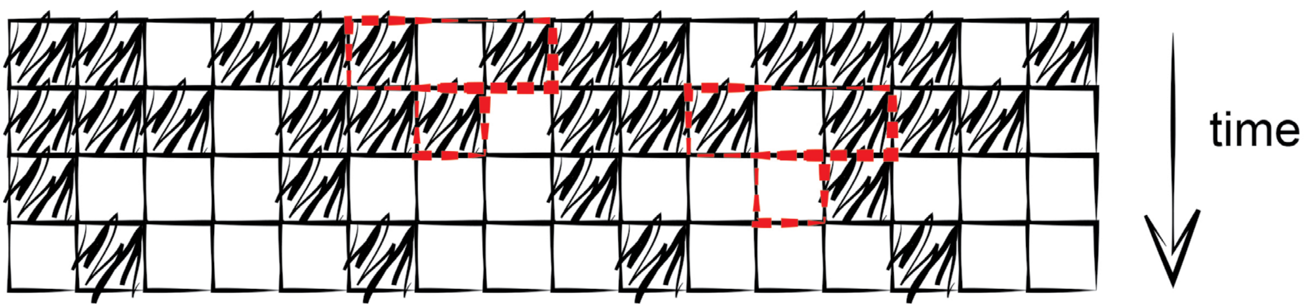

Figure 9. Hypothesized development for t = 1, 2, 3, 4 of a Pied-de-poule of type N = 1 on a one-dimensional grid of 16 cells. The two dashed environments indicate that there is a difficulty for a formal rule to produce the pattern if we would adopt two states 0 (white) and 1 (black) only.

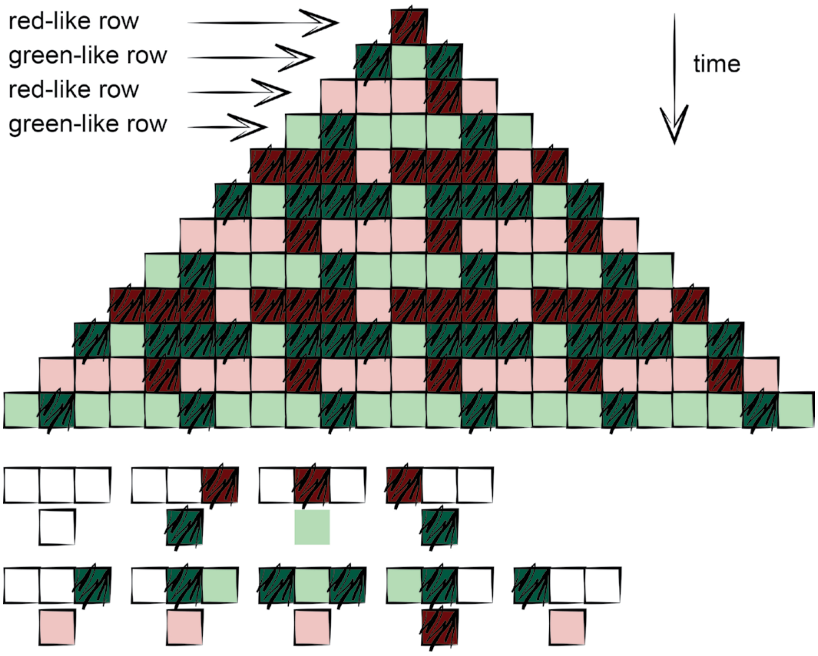

Figure 10. First 12 rows of a Pied-de-poule development (left) and 9 maplets which are sufficient for development of the first two rows (*t* = 2 and *t* = 3) from the initial row (*t* = 1).

Formally the cellular automata A_L, one for each positive L, are defined by:

- the circular grid of L cells,
- the state set $Q = \{-2, -1, 0, 1, 2\}$,
- the environment structure defined by $r = 1$,
- the rule of the 35(+default) maplets:
  (growth) {0, 0, 0}→0, {0, 0, 1}→2, {0, 1, 0}→ −2, {1, 0, 0}→2, {0, 0, 2}→ −1, {0, 2,−2}→ −1, {−2, 2, 0}→1, {2, 0, 0}→ −1, {0, 0,−1}→2, {0,−1,−1}→2, {1,−1, 0}→2, {−1, 0, 0}→ −2, {0, 0,−2}→1, {0,−2, 2}→1, {2,−2, 0}→ −1, {−2, 0, 0}→1, {0, 1, 1}→ −2, {−1, 1, 0}→ −2, {−2, 0,−2}→1. (sustaining) {2,−2, 2}→ −1, {−1, −1,−1}→ −2, {−1, −1, 1}→ −2, {−1, 1,−1}→ −2, {−2, 2,−2}→1, {2,−2,−2}→ −1, {−2, −2,−2}→1, {−2, −2, 2}→1, {1, 1, 1}→2, {1, 1,−1}→2, {1,−1, 1}→2, {−1, 1, 1}→ −2, {−2, 2, 2}→1, {2, 2, 2}→ −1, {2, 2,−2}→1, {1,−1,−1}→2 and (default) { _ ,_ , _ }→0.

Generating Patterns

The automata A_L for $L > 4$ give an answer to the question whether a cellular automaton can generate Pied-de-poule patterns. The answer is *yes*. But is it already interesting? Is there an emerging complexity? In Figure 11 we see a typical emerging behavior. The initial sparse random grid contains dark-red (1) and dark-green (2) seeds, generated according to a probability distribution $P(1) = P(2) = 1/24$ and $P(0) = 11/12$. We show the evolution for $t = 1, 2, \ldots 2048$. Similar patterns appear for different initial grids (produced with the same probability distribution). This is precisely what Wolfram (1984) claims: "Cellular automata may also be characterized by the stability or predictability of their behavior under small perturbations in initial configurations" (p. 420). In Figure 13 we see what happens in more detail.

The drip stripes of Figure 11 are an emerging phenomenon. They appear random, but on average they drift slightly to the left with a characteristic angle of about 5°. This is an emerging property. The emergent behavior of the drip stripes is largely independent of the initial configuration (with few exceptions such as the initially blank grid, which of course leads to different behavior). The drip stripes wobble a bit and occasionally two of them meet (and then both stop). Although eventually the automaton will evolve toward a repetitive state for most initial configurations, the cancelling-out of the last two drip stripes usually happens at a high t values. For example, even for a modest grid length $L = 256$ we find that it typically takes between 10,000 and 100,000 time steps for the drip stripes to disappear (sometimes even more). Wolfram defines a class II automaton as an automaton which rapidly converges to a repetitive or stable state; our automaton does converge, but not rapidly. Initially, the grid is crowded by drip stripes and they meet easily and then stop. But the last two drip stripes can be running in parallel for a significant vertical distance (time). The fewer drip stripes remain, the longer it takes for them to disappear.

To gain a better understanding of what happens here, we tested $A_L^{sustain}$ where $A_L^{sustain}$ is like A_L but having the growth maplets removed (only retaining the *sustaining* maplets). We tested $A_L^{sustain}$ on all 4 initial non-zero grids for L = 1, 2, 3, 4, 6, 8, 16 and found that, for each L, there exist one or more initial grids which produce sustainable patterns without invocation of the default rule. These patterns are fixed points of the automaton (and are also fixed points of larger grids). A few of them are shown in Figure 12 for $t = 1, \ldots 9$. There are infinitely many of such fixed points which can be classified according to the smallest horizontal translation that leaves them invariant (there are in essence only four which are invariant under a translation of four cells: the top row in Figure 12. The fourth pattern in the top row in Figure 12 is the real Pied-de-poule pattern. The others are a kind of Pied-de-poule lookalikes (we call all of those faux Pied-de-poules).

In Figure 13 we zoom in to see more detail and explain what is happening: The initial seed gives rise to expanding and self-sustaining areas, where each area is a classic Pied-de-poule, a vertical or slanted faux Pied-de-poule or some other faux Pied-de-poule. But there are not enough maplets in the rule to repair the effects of colliding areas. Since the 35 maplets have been obtained from a single dark(-red) seed only, the only collision which is properly handled is the collision after wrap-around (on the circular grid) when two identical Pied-de-poule patterns grown from seeds separated by a multiple of 4, collide. We could say that the (faux) Pied-de-poule areas collide, like tectonic plates. At the plate boundaries, there are hardly any applicable maplets, which means that, by default, blank space gets introduced. Once there is a combination of non-zero positions and blank space next to it, the growth maplets do their work again and feed a non-zero pattern into the plate. Occasionally, two plates are reconciled, otherwise a boundary region persists. Zooming out, we recognize the collision areas as the drip stripes of Figure 11.

But there is a more subtle effect, which is hardly noticeable in Figure 11. We need to zoom in as in Figure 14. Inside the tectonic plates there are diagonal zones, separated by transitions, such as the two zones and the transition highlighted by the zoom lens. The transitions run under an angle of 45°. Above the transition there is a proper Pied-de-poule, below the transition a kind of faux Pied-de-poule. The transition happens without blanks. It originates at the plate boundary and moves rightwards. There are many of these subtle transitions. Even after disappearance of the drip stripes, the diagonal zones and their transitions live on.

We claim that the three key phenomena of complexity theory, viz. emergence, transition, and resilience (Vermeer, 2014) all appear in the behavior of the designed automaton. We also played with similar automata obtained by adding extra random

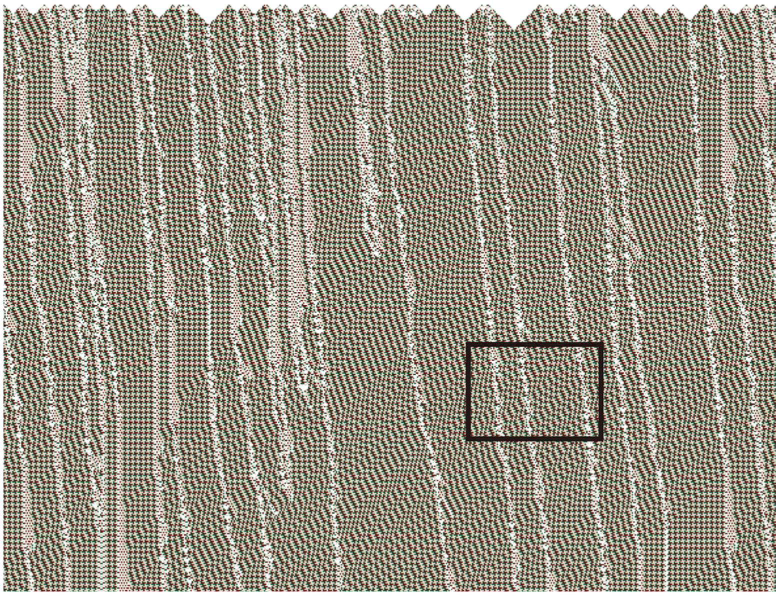

Figure 11. Running the automaton A2048 on a random grid with sparse dark-red and dark-green seeds.

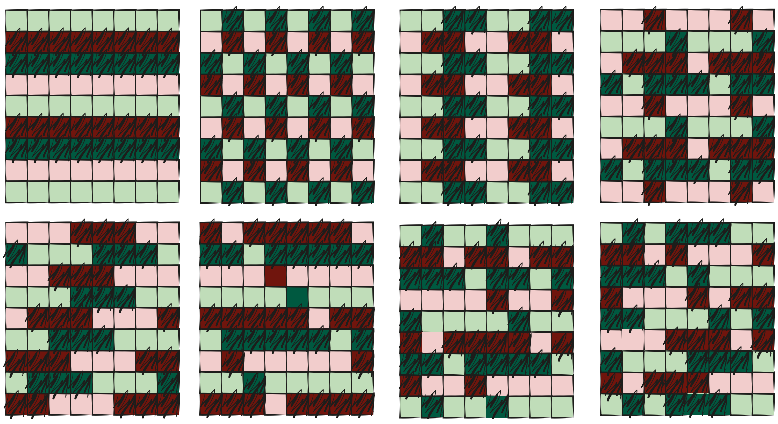

Figure 12. Eight of the fixed points of the automaton. Top row left to right: zebra pattern, vertical zigzag, blocks and Pied-de-poule (invariant under horizontal translations of 1, 2, 4, and 4 cells, respectively). Second row left to right: diagonal zigzag, elongated Pied-de-poule, another elongated/mixed faux Pied-de-poule and complicated faux Pied-de-poule (invariant under 6, 6, 8, 8).

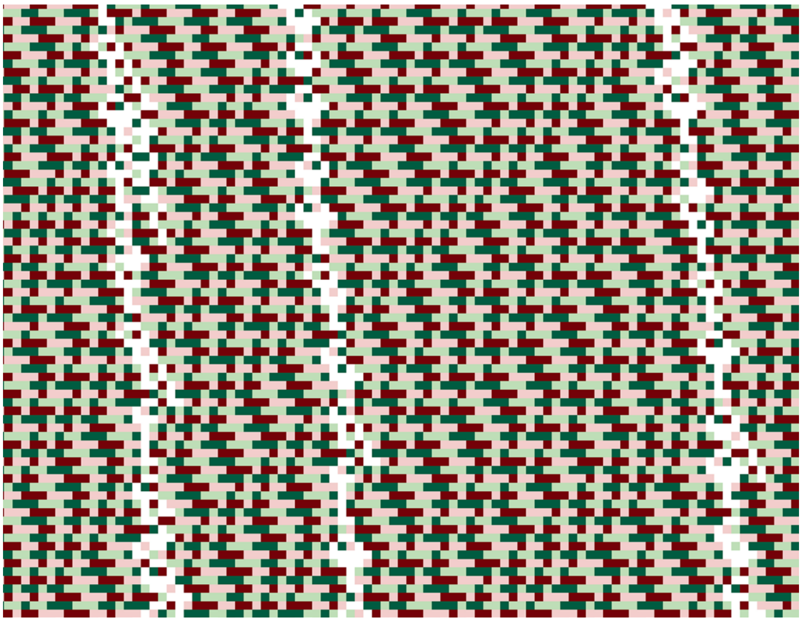

Figure 13. Zooming in on the pattern generated by A2048.

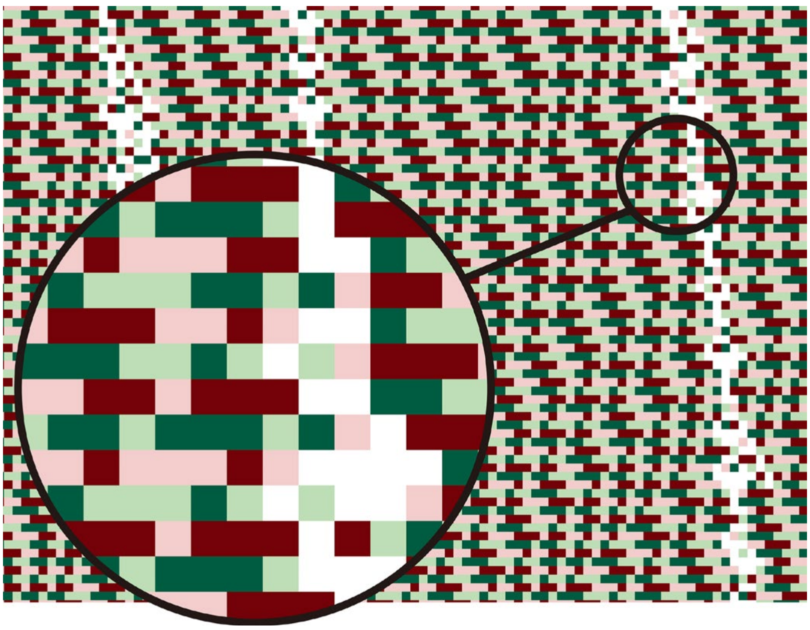

Figure 14. Subtle transitions moving rightwards through the tectonic plates.

maplets (typically a few dozen) and found that a wide variety of effects could be obtained. In the presence of extra random maplets, the Pied-de-poules and faux Pied-de-poules keep on re-appearing. Three possibilities will be given later in Figure 15. We do not provide a full survey of such possibilities, because the number of possible rules (for $r = 1$ and $k = 5$) is very large, about 2.3 × 1087.

There is a family of Pied-de-poule patterns (Feijs, 2012), one for each integer $N > 0$. The Pied-de-poule pattern of Figure 9 is $N = 1$, just the simplest of the family (which is why sometimes it is called puppytooth). We found that it is possible to develop cellular automata for larger N as well (provided more colors are used). Moreover, one-dimensional automata are just the simplest case in a whole range of possibilities: two-dimensional, three-dimensional, etc. Perhaps some of the techniques from Section *Designing an Automaton for Pied-de-poule* could be generalized to two dimensions, but working two dimensions is more difficult since the number of possible rules is huge and behavior is hard to predict. 2D is better for dynamic effects, such as Conway's Life, see Gardner (1970). But for choosing the pattern to be deployed now we stay with the simplest Pied-de-poule and the lowest dimension for the automata (most of the dynamical features of cellular automata can be found in the study of the one-dimensional case). In view of the *less is more* principle, we feel the message becomes stronger if we use the simplest cases. For the remainder of this project, we work with Pied-de-poule of type $N = 1$ and in one dimension.

In Figure 15 we show the three patterns chosen as the basis for weaving the fabric. The leftmost pattern is obtained by the same 35 maplets discussed in Section *Designing an Automaton for Pied-de-poule*. Recall that these were derived automatically as the minimal automaton which generates the Pied-de-poule of type $N = 1$. The grid size is 1072, the number of steps is 526. Compared to the previous images, we have turned 90° so in Figure 15 the automaton evolves from left to right. Each pattern is repeated twice in the vertical direction, which is the direction of warp (according to the way we shall actually weave it in Jacquard).

For the second and third pattern we have added another 35 maplets, which were randomly generated. We went through this process of randomly generating maplets a few dozen times. Most of the cellular automata thus obtained produce drip-line patterns. However, about ten percent of them behave differently and we picked two such automata which we felt interesting. The second pattern has grid size 536 and 263 steps. The third pattern has grid size 268 and 132 steps (note its characteristic lines at angles of 45° and 26.57°).

These three patterns are woven with different scales of magnification so that each ends up as 49 × 50 cm. Together with a representation of the maplets and brand labels this makes a weaving of 150 cm width and 50 cm length. Thus, we have 16 meters woven by EE Labels in Heeze, The Netherlands, a company specialized in woven labels and other products of the very best quality to a wide range of leading global brands.

The translation of the proposed representational 2D graphical pattern, generated through cellular automata simulation, into woven fabric is described next. We present our choices regarding the equipment, the warp and the weft.

- Equipment: Jacquard loom (Gandhi, 2012). Although most Pied-de-poule fabrics are still produced on traditional looms, we have chosen for a modern production technique: a computer-controlled Jacquard loom. A Jacquard loom allows every individual warp yarn to be lifted or lowered when the weft passes. Unlike in classic Pied-de-poule weaving, the cells no longer have a one-to-one mapping to the warp-weft crossings. The loom has a much higher granularity than the cellular automata grids. The mapping was fine-tuned by a specialist at EE labels, using proprietary conversion software.
- Warp: black. All warp yarns are black, which is one of the default machine configurations. Setting up a Jacquard loom with thousands of warp yarns is a huge task, so it is much more efficient to work with a standardized warp, in our case black. All other colors, including white, are realized by the weft. For realizing the leftmost pattern of Figure 15 (the pattern with the smallest cells), the implementation of one cell takes 6 warp yarns, for the center pattern 12 warp yarns and for the rightmost 24.
- Weft: five colors. Inside a dark-green cell of the leftmost pattern of Figure 15 for example, the dark-green weft is on top almost everywhere, with a float length of six, sometimes

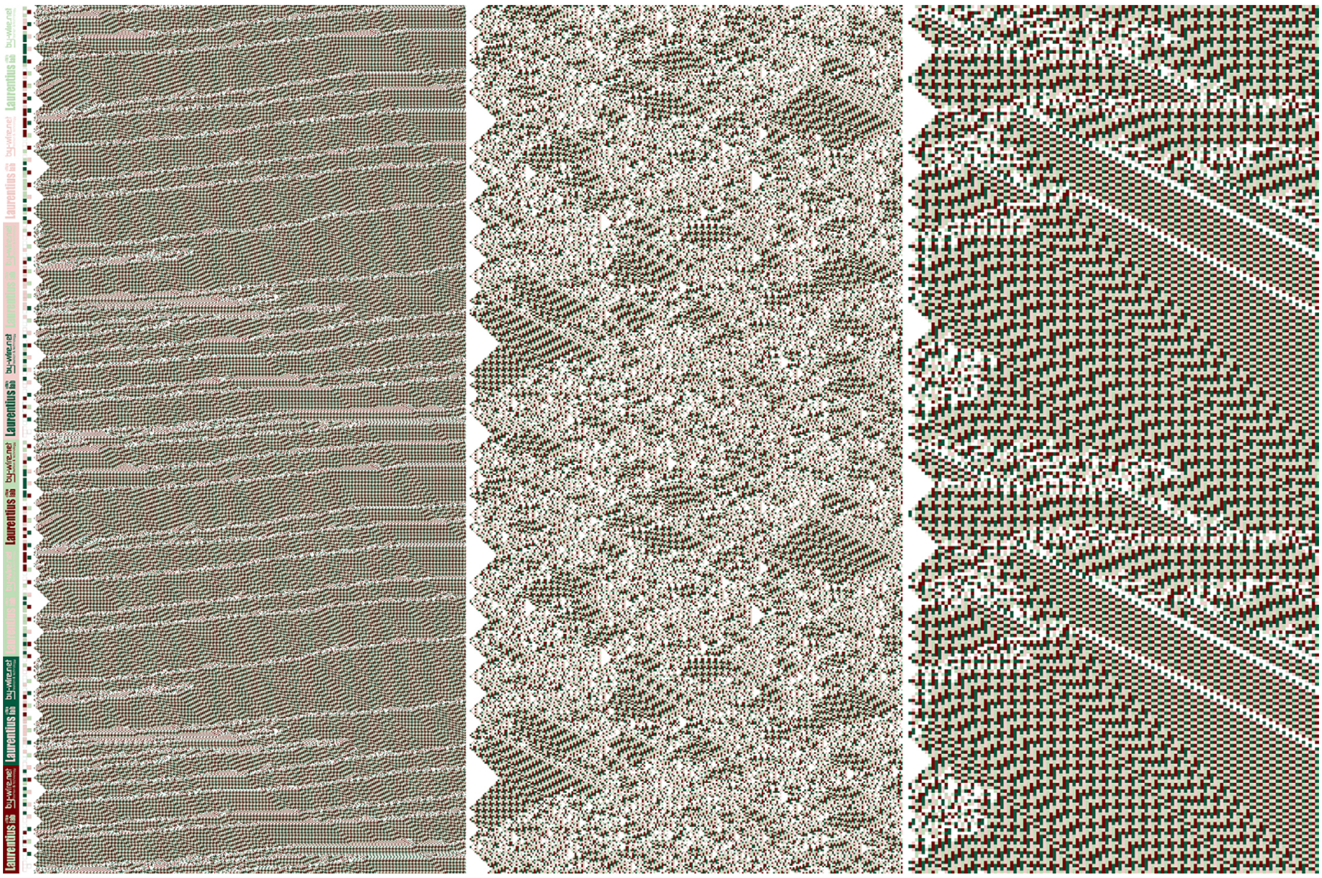

Figure 15. Three different generated patterns.

two or three. The same holds for the other colors (dark-red, greeny, pinky and white). Consecutive weft yarns are densely pushed against each other, using 56 shots (weft yarns) per centimeter. The weft yarns are much thicker than the warp yarns; therefore the black warp is hardly visible. For the center and rightmost patterns of Figure 15, a weft-faced unbalanced twill binding is used (five over, one under, like satin). The backside of the fabric appears as a seemingly random mix of long floats in all five colors.

The details of the weaving can be seen in the microscopic image in Figure 16, where the diameter of the image corresponds to 1 cm of fabric. The image shows a sample of the leftmost pattern of Figure 15. The warp yarns appear vertically, the weft horizontally. In this sample each cell is implemented by two or three weft yarns, but these interlock with the weft of adjacent cells, so there are on average five shots per cell.

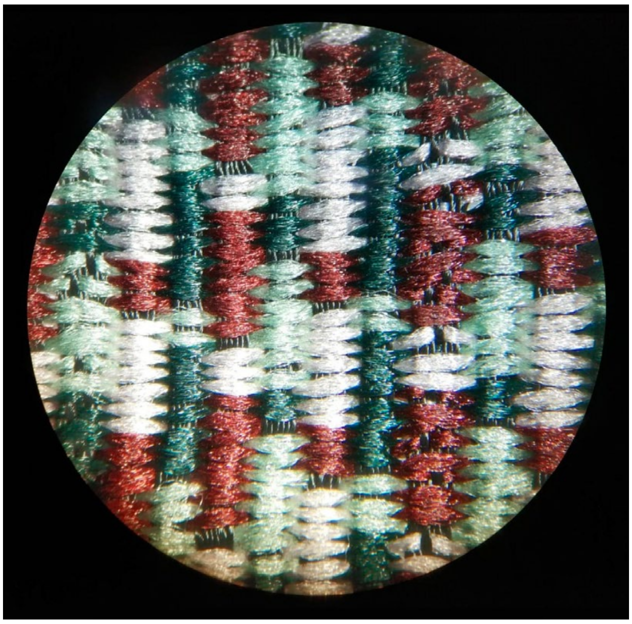

Figure 16. Microscopic image of one of the computer-generated cellular automata patterns realized as a Jacquard-woven fabric.

The Process

The process is presented in Figure 17, in which we explain how the coding process is integrated with the fashion design processes. There were many iterations in the coding phase and multi-disciplinary cooperation in the overlapping weaving, design and construction phases. The knowledge was distributed over different persons, roughly speaking Feijs being most active in the *creative math research* (gray colored area) yet having specific knowledge on weaving and having access to a network with expertise on the complexity of the fashion system. Toeters is an expert in fashion and fashion technology, which includes part of the gray area and most of the fashion-related areas. Additional weaving expertise and all of the weaving production was contributed by EE labels.

There were many iterations inside the *creative math research* track, which still continued during the interaction with the weaving company (track *fabric design and weaving*) and during the initial sketching of the garments (track *garments design & implementation*). The other tracks were more linear. Once the actual weaving had begun, the cellular automata were frozen. Final details of the garments were added during garment design and construction with two extra aims: (1) to be innovative (newness, e.g., magnetic zipper instead of traditional closure) and (2) to follow contemporary trends (desirability, such as the bomber jacket, which is a must-have in today's collections).

The project appears somewhat skewed towards the coding side. This is because we told the story about a new understanding of complexity and emergent behavior through the fabric's pattern. Yet we went all the way, up to and including a collection (most other coding-based projects in the field stop having achieved a single item such as a ring, a hat or a scarf).

The Collection

We have realized a small collection based on the work of the previous sections. For the development of the design, we took into account that there is a fine balance between the feeling that garments fit in current looks, them being old fashioned or very innovative (see section *Fashion as a Complex System*). We aim to position the garments as fitting in current society but just a little innovative when zooming in.

As a first context, we chose a mathematical art exhibition (Toeters & Feijs, 2017). Pattern design, colors, weaving structure and presentation context are thus clear. We deal with many connotations to the past (Pied-de-poule, familiar colors), but also introduce new aspects (complexity theory and cellular automata in fashion). As designers, we are in constant dialogue with all concerned aspects. The shapes of the items and the garment details are last to be defined: the last chance, on a product level, to compensate on newness and to create desirability.

In the item definition, we used very classic recognizable and well-accepted shapes like the man's jacket and shirt. In the jacket as shown in Figure 18, top row right, we adjusted minor details like the side seam position, the connected back panel at the top part and the way to enter the pockets (from above). The jacket shows the left pattern from Figure 15. The shirt in Figure 18, second row left, shows the center pattern from Figure 15. These items look very familiar and are well-accepted.

As a very current and fashionable item, we chose a bomber jacket which can be found on almost every 2017 catwalk as well as on the street. Ours, as shown in Figure 18, second row center, is with a twist, as we combine different patterns in one item. It has no side seams so that the pattern continues around the body and shows the weaving method. There is no lining used, and the garment is prepared to be worn inside out as well. We chose for this approach because the backside of the woven material is also very interesting.

As another contemporary and fashionable item we chose for an A-line dress (as shown in Figure 18, second row right) that shows the right pattern of Figure 15. As a more innovative item

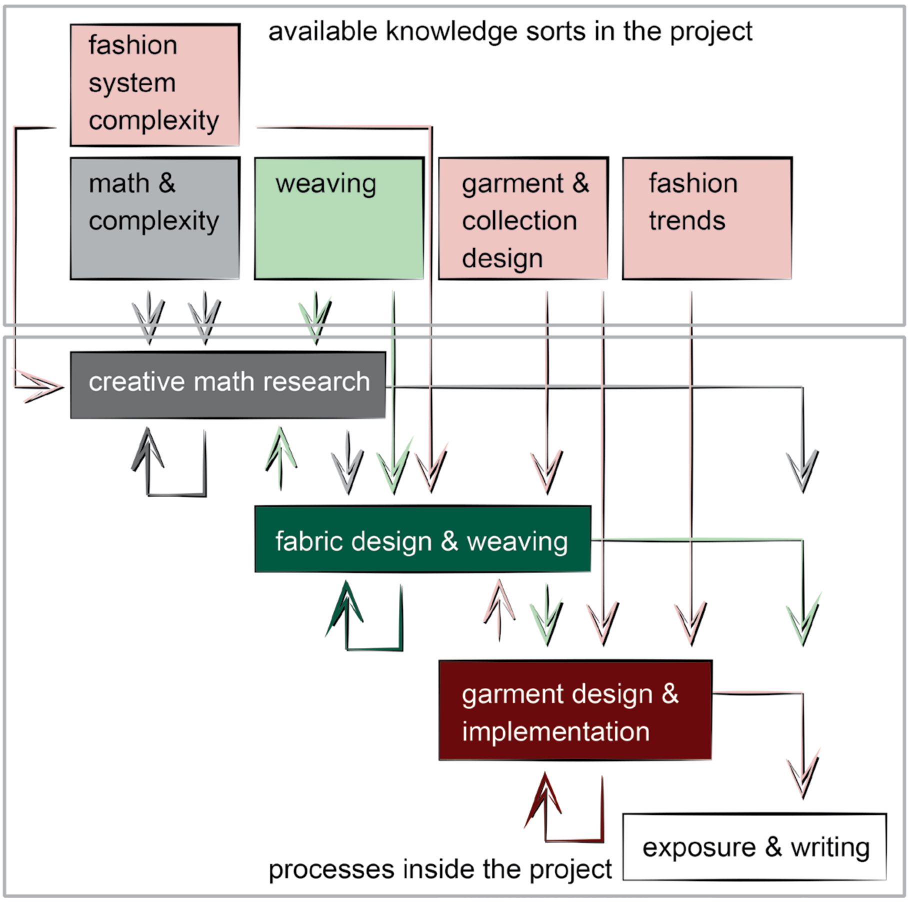

Figure 17. Overview of the design process with feedforward and feedback between the different tracks.

Figure 18. The six designed garments based on the algorithmically generated patterns.
The material is Jacquard-woven polyester. Photos by Robin van der Schaft, styling by Maaike Staal (© Marina Toeters).

we developed a strapless jumpsuit (onesie) as shown in Figure 19 rightmost and in Figure 18 top row, left. This item is designed around the pattern and uses the total width of the fabric once. The logo like just above the breasts is followed by all three patterns all the way to the ground. The used circumference is exactly 98 cm so that the pattern continues over the only seam used in the top part. The horizontal white areas between the three patterns introduce some visually exciting effects towards the feeling of gravity within the garment. Most likely this is because of the position of the lines (yet it happened by serendipity). The size is extremely long to emphasize the full fabric width.

The sleeveless top, as shown in Figure 18 top row, center, is an eclectic patchwork of all leftover pieces. To address innovativeness, even more, we introduced a magnet zipper in this top. At the detail level we have innovations such as a zipper in the classical man-shirt and 3D printed magnet closures in the man's jacket. More details can be seen in Figure 19. Left in Figure 19, from top to down, we see the pocket on the man's jacket, the magnetic zipper and the eclectic patchwork of the sleeveless top, the bomber jacket inside-out and another detail of the man's jacket, viz. the 3D printed magnet closure. Right in Figure 19 is the strapless jumpsuit (onesie).

Figure 19. Garment details: Pocket, magnet zipper in eclectic patchwork, bomber inside-out and 3D printed magnet closure to the left from top to down and a full strapless jumpsuit to the right (© Marina Toeters).

When making decisions about which patterns to be used for which garments, we tried to reflect some of the fashion-as-a-complex-system insights discussed in section *Fashion as a Complex System*: The long drip lines in the man's jacket in Figure 18, top row foremost right, can be read as a reference to the slow change in men's wear, where the basic design of men's jackets is very stable over decades, although there are tiny changes every season; in essence, this is caused by the bandwagon effect mentioned in section *Fashion as a Complex System*: many men just do not want to really stand out by their garment choice. The waves of zones of the foremost right pattern of Figure 15 can be seen as analogous to trends (discussed in section *Fashion as a Complex System*). They are made visible in the A-line dress, as shown in Figure 18, second row right , as the trends in women's wear are more pronounced than in men's wear. Referring to Laurell's fashion spheres (Laurell, 2016), the sleeveless top, as shown in Figure 18, top row center, with all the eclectic patches reminds of the fact that there are multiple factors contributing to the complexity of the fashion system.

Two of the outfits shown in Figures 18 and 19 were deployed by the staff of EE Labels as part of the exhibit EE Exclusives during the Milano Design week 2017 at Palazzo Francesco Turati. We contacted and received positive reactions from Prof. Dr. Jos Baeten, Professor of Mathematics and director of the Center for Mathematics and Computer Science in Amsterdam, and Prof. Dr. Sjoerd Verduyn Lunel, Professor of Mathematics, Secretary of the European Mathematical Society, Chair of the Dutch Platform for Mathematics and Director of Research of the Mathematical Institute of Utrecht University, who were very enthusiastic about wearing the outfits and acted as the models for the photoshoot (which is in the catalogue of the 2017 Bridges Mathematical Art exhibition; Toeters & Feijs, 2017) and in the short paper (Feijs & Toeters, 2017). The American Mathematical Society noticed the exhibit, twittering *Fabulous fabrics: Cellular automaton-based fashion collection* and the professional Wolfram Blog wrote about this *applying new technology to traditional weaving patterns*. The shirt as shown in Figure 18, second row foremost left, was presented in the *UNiD Magazine* No 32 of the LUCID design study association. The complete collection was on the catwalk of the first Bridges mathematical fashion show in Stockholm and was exhibited during Dutch Design Week in 2018. More venues will be addressed in 2019.

Results and Discussion

We summarize our results and claims from five perspectives: semiotics, multi-disciplinarity, generative design in fashion, complexity theory and the future.

Semiotic perspective: The role of designers is twofold. First they want to give form to new products which serve practical needs and are satisfying with respect to function, comfort, aesthetics and sustainability. At the same time, designers use emerging technologies and semiotic tools to convey messages about what is going on in the world and point to new possibilities. The work of the present paper falls in the latter category: the designed garments convey the message that there is a new understanding of complexity which is relevant for many disciplines, including fashion and design itself. The garments encode the message in an iconic way: the patterns appear complex and lack the regularity of traditional weaving and printing (iconic in a Peircean sense; Chandler, 2003; Feijs & Meinel, 2005; Peirce, 1931). This complexity can be easily observed by anyone, without prior knowledge. *Fashionable* is encoded by the puppytooth Pied-de-poule which is visible as a recurrent sub-pattern and thus resembles other Pied-de-poules in fashion. Additionally, the messages of *contemporary*, *innovative,* and (again) *fashionable* are coded in a multitude of ways, as explained in section *The collection*. The garments also code the message that there is a new understanding of complexity in a two-step manner: whoever has seen a glimpse of Conway's and Wolfram's works will immediately recognize the cellular automata and hence, indirectly, modern complexity theory. But modern complexity theory is not well-known among the general public and, therefore, discussing the garments offers people an opportunity to expand their semantic code, as described by Eco's ratio-difficilis (Eco, 1979).

Multi-disciplinarity perspective: We claim that programming is a new craft which is essential for a range of emerging new aesthetic possibilities in design and for developing new product semantics. Programming is not only needed for the behavior and embedded software of electronics in interactive wearables, it is also a powerful tool for choosing aesthetic qualities and coding messages (coding in the semiotic sense: Eco, 1979). This is what this project demonstrates. In this project we worked with Mathematica (version 10), one of the most powerful mathematical tools available. More and more, the embedded software approach and the generative design approach will be mixed, as in Drapely-o-lightment (Feijs & Toeters, 2015a), Solemaker (Feijs, Nachtigall & Tomico, 2016) and Bedtime Stories (Kuusk, Wensveen & Tomico, 2014). Previous case studies have demonstrated the potential of our kind of cross-disciplinary cooperation. Our earlier projects included Drapely-o-lightment (a new aesthetics and at the same time an exploration into the interplay of hard and soft materials while integrating electronics into garments; Feijs & Toeters, 2015a) and Pied-de-pulse (pushing the frontier of soft actuators by embroidering them into the fabric and at the same time inscribing a reference to the Pied-de-poule pattern; Feijs & Toeters, 2016). These projects would be impossible without the cross-disciplinary cooperation of a person with a programming/math background and a person with a design/fashion/innovator profile.

Generative design in fashion perspective: As described in sections *Designing automata for Pied-de-poule* and *Generating patterns* it should be noted that the proposed approach does not deliver a fixed pattern. As Wolfram (2010) puts it: "In a sense, we can use the computational universe to get mass customized creativity". It is a technology for generating unique patterns with considerable freedom for mass customization. It works like parametric design—the choice of rule determines the complexity of the pattern. We claim that this generative design perspective can go hand in hand with preservation and revitalization of traditional cultural themes and craft-related themes (in this case Pied-de-poule) rendering them by new computer-controlled production technologies. Kuusk et al. (2014) showed this for digital embroidery, we used computer-Jacquard to translate our patterns into woven garments.

Complexity theory perspective: We are very attracted to this complexity theory. The fashion system is an example of a Complex Adaptive System. In this article we represented some of our understanding via the advanced programming and production tools which we have at hand in the widely accepted medium of garments and fashion. The complexity of the fashion system is not unraveled, but we claim that we made a contribution, not only by this paper, but also through expositions and creating awareness of the fact that the fashion system is a complex adaptive system.

Future perspective: Everyone wears garments, can we start using them as pieces for educating new complexities? This is what we hope to research further in future.

Acknowledgements

We like to thank the editors and the anonymous reviewers of *IJDesign* for their helpful feedback. We thank Laura Tedrekull, Nadeřda Jcrilova, Ruth-Gertrud Rass, Reti-Lii Ôunapuu, Robin Matulessy, Marc Evers, Joost Jansen, Frank Delbressine, Robin van der Schaft, Maaike Staal, Nicole Gruithuijzen and Troy Nachtigall for their help and support during this project.

References

1. Ahmed, A. G. M. (2014). Modular duotone weaving design. In G. Greenfield, G. Hart, & R. Sarhangi (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 27-34). Phoenix, AZ: Tessellations.
2. Banda, P., Caughman, J., & Pospichal, J. (2015). Configuration symmetry and performance upper bound of one-dimensional cellular automata for the leader election problem. *Journal of Cellular Automata, 10*(1-2), 1-21.
3. Bezbradica, M., Crane, M., & Ruskin, H. J. (2016). Applications of high performance algorithms to large scale cellular automata frameworks used in pharmaceutical modelling. *Journal of Cellular Automata*, *11*(1), 21-45.
4. Bin, H., & Zhang, D. (2006). Cellular-automata based qualitative simulation for nonprofit group behavior. *Journal of Artificial Societies and Social Simulation, 10*(1). Retrieved from http://jasss.soc.surrey.ac.uk/10/1/1.html.
5. Cagliano, A. C., DeMarco, A., Rafele, C., & Volpe, S. (2011). Using system dynamics in ware- house management: A fastfashion case study. *Journal of Manufacturing Technology Management*, *22*(2), 171-188.
6. Castillo, F., Toledo, B. A., Muñoz, V., Rogan, J., Zarama, R., Penagos, J. F., & Valdivia, J. A. (2016). Spatiotemporal complexity of a city traffic jam. *Journal of Cellular Automata, 11*(5-6), 381-398.
7. Chandler, D. (2003). *Semiotics, the basics*. London, UK: Routledge.
8. Coleman, M. (2012). *Pretty smart textiles* (Exhibition). Retrieved from http://prettysmarttextiles.com/exhibition2012belgium/.
9. Crane, D. (1999). Diffusion models and fashion: A reassessment. *The Annals of the American Academy of Political and Social Science, 566*(1), 13-24.
10. De Comité, F. (2014). Cardioidal variations. In *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 349-352). Phoenix, AZ: Tessellations.
11. Eco, U. (1979). *A theory of semiotics*. Bloomington, IN: Indiana University Press.
12. Edelkoort, L. (2015). *Anti fashion manifesto: Ten reasons why the fashion system is obsolete*. Paris, France: Trend Union.
13. Feiereisen, S. (2013). Viktor & Rolf go all black for haute couture fall 2013. *The Fashionspot, News & runway*. www.thefashionspot.com/runway-news/310985-viktor-rolf-haute-couture-fall-2013/ (retrieved 3-12-2018).
14. Feijs, L. M. G., & Meinel, F. (2005). A formal approach to product semantics with an application to sustainable design. *Design Issues, 21*(3), 67-81.
15. Feijs, L. M. G. (2012). Geometry and computation of houndstooth (Pied-de-poule). In R. Bosch, D. McKenna, & R. Sarhangi (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 299-306). Phoenix, AZ: Tessellations.
16. Feijs, L. M. G., & Toeters, M. J. (2013). Constructing and applying the fractal pied de poule (houndstooth). In G. Hart & R. Sarhangi (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 429-432). Phoenix, AZ: Tessellations.
17. Feijs, L. M. G., & Toeters, M. J. (2015a). Drapely-o-lightment: An algorithmic approach to designing for drapability in an e-textile garment. *Leonardo, 48*(3), 226-234.
18. Feijs, L. M. G., & Toeters, M. J. (2015b). A novel line fractal pied de poule (houndstooth). In K. Delp, C. S. Kaplan, D. McKenna, & R. Sarhangi (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 223-230). Phoenix, AZ: Tessellations.
19. Feijs, L. M. G., & Toeters, M. J. (2016). Pied-de-pulse: sphere packing and Pied-de-poule (houndstooth). In E. Torrence, B. Torrence, C. Séquin, D. McKenna, K. Fenyvesi, & R. Sarhangi (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 415-418). Phoenix, AZ: Tessellations.
20. Feijs, L. M. G., Nachtigall, T., & Tomico, O. (2016). Sole maker: Towards ultra- personalised shoe design using voronoi diagrams and 3D printing. In *Proceedings of the Fabrication and Sculpting Event* (pp. 31-40). Retrieved from https://pure.tue.nl/ws/portalfiles/portal/41976954/1.pdf.
21. Feijs, L. M. G., & Toeters, M. J. (2017). A Cellular Automaton for Pied-de-poule (houndstooth). In D. Swart, C. H. Séquin, & K. Fenyvesi, (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 403-406). Phoenix, AZ: Tessellations.
22. Franklin, C. (2012). *Fashion: The ultimate book of costume and style*. London, UK: Dorling Kindersley.

23. Frei, K. M. (2009). News on the geographical origin of the Gerum cloak's raw material. *Fornvnnen Journal of Swedish Antiquarian Research, 104*(4), 313-315.
24. Fredriksson, C. (2008). Trend analysis for fashionistas only? In C. Fredriksson & H. Jönsson (Eds.), *Job, etnologisk skriftserie etnologiska institutionen* (pp. 83-90). Lunds, Sweden: Lunds university.
25. Gandhi, K. L. (2012). The fundamentals of weaving technology. In K. L. Gandhi (Ed.), *Woven textiles: Principles, technologies and applications* (pp. 117-160). Cambridge, UK: Woodhead Publishing.
26. Gardner, M. (1970). Mathematical games, the fantastic combinations of John Conway's new solitaire game "life". *Scientific American 223* (October 1970), 120-123.
27. Guan, J., Wang, K., & Chen, F. (2016). Self-organization phenomena in an evacuation flow. *Journal of Cellular Automata, 11*(5-6), 461-473.
28. Holden, J., & Holden, L. (2016). Modeling braids, cables, and weaves with stranded cellular automata. In E. Torrence, B. Torrence, C. Séquin, D. McKenna, K. Fenyvesi, & R. Sarhangi (Eds.), *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (127-134). Phoenix, AZ: Tessellations.
29. Gennert Jakobsson, J. (2018). *Who's tooth? Houndstooth! An investigation about how to use houndstooth pattern to generate form and surface with a cut-and-weave method* (BA Thesis), The Swedish School of Textiles, University of Borås, Borås, Sweden.
30. Kim, S., & Ahn, I. (Nov. 12, 2015). *Impact of macro-economic factors on the hemline cycles*. Paper presented at the Conference of the International Textile and Apparel Association, Iowa State University, Ames, IA.
31. Kosztowny, A. J. (2015). *The preservation of physical fashion forecasts* (Master thesis). University of California, Los Angeles, CA.
32. Krippendorff, K. (1989). *Product semantics: A triangulation and four design theories.* Paper presented at the Conference on Product Semantic, University of Industrial Arts, Helsinki, Finland.
33. Kuhn, R., & Minuzzi, R. F. B. (2015). The 3D printing's panorama in fashion design. *Moda Documenta: Museu, Memoria e Design, 11*(1), 1-12.
34. Kuusk, K., Wensveen, S. A. G., & Tomico, O. (Nov. 26, 2014). *Crafting qualities in designing QR-coded embroidery and bedtime stories*. Paper presented at the 5th Conference on the Art of Research, Helsinki, Finland.
35. Langton, C. G. (1986). Studying artificial life with cellular automata. *Physica D: Nonlinear Phenomena, 22*(1-3), 120-149.
36. Laurell, C. (2016). Fashion spheres from a systemic to a sphereological perspective of fashion. *Journal of Fashion Marketing and Management, 20*(4), 520-530.
37. Law, K. M., Zhang, Z. M., & Leung, C. -S. (2004). Fashion change and fashion consumption: The chaotic perspective. *Journal of Fashion Marketing and Management, 8*(4), 362-374.
38. Ligenza, G. (2015). Fashion in 3D. *Feel Magazin, 1*(1), 44-47.
39. Matsumoto, E. A., Segerman, H., & Serriere, F. (2018). Möbius cellular automata scarves. In *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 523-526). Phoenix, AZ: Tessellations.
40. McBurney, S. (2009). On constructing a virtual loom. In *Proceedings of the Conference of Bridges: Mathematics, Music, Art, Architecture, Culture* (pp. 287-292). Phoenix, AZ: Tessellations.
41. Miller, C. M., McIntyre, S. H., & Mantrala, M. K. (1993). Toward formalizing fashion theory. *Journal of Marketing Research, 30*(2), 142-156.
42. Nachtigall, T. (2017). *The life of fashion trends: Speculating on how fashion trends happen using complex adaptive systems.* Retrieved from https://thelifeoffashiontrends.wordpress.com/.
43. Peirce, C. S. (1931). *Collected papers of Charles Sanders Peirce* (vol. 8). Cambridge, MA: Harvard University Press.
44. Quinn, B. (2002). A Note: Hussein Chalayan, fashion and technology. *Fashion Theory, 6*(4), 359-368.
45. Rackham, M. (2009). Coded cloth: A 21st-century revolution in art, fashion and design. *Leonardo, 42*(5), 386-388.
46. Schiff, J. L. (2011). *Cellular automata: A discrete view of the world*. Hoboken, NJ: Wiley.
47. Shiffman, D., Fry, S., & Marsh, Z. (2012). *The nature of code*. New York, NY: Daniel Shiffman.
48. Tenthof van Noorden, L. S., Feijs, L. M. G., Toeters, M., Hu, J., Liu, J., & Feijs, K. (2013). This fits me and warp knit fractal. In R. Fathauer & N. Selikhoff (Eds.), *Bridges Seoul art exhibition catalog*. Phoenix, AZ: Tessellations.
49. Toeters M. J., & Feijs, L. M. G. (July 27, 2017). *Cellular automaton-based mini fashion collection*. Artwork exhibited at the Conference of Bridges, University of Waterloo, Waterloo, Canada.
50. Van Kessel, A. (2013). Technological haute couture. *Cursor 56*(1), 26-27.
51. Vermeer, B. (Ed.). (2014). *Grip on complexity. How manageable are complex systems? Directions for future complexity research*. The Hague, The Netherlands: Netherlands Organization for Scientific Research (NWO).
52. Wilson, J. (2012). Woven structures and their characteristics. In K. L. Gandhi (Ed.), *Woven textiles: Principles, technologies and applications* (pp. 163-204). Cambridge, UK: Woodhead Publishing.
53. Wolfram, S. (1984). *Cellular automata as models of complexity*. *Nature, 311*(5985). 419-424.
54. Wolfram, S. (2002). *A new kind of science*. Champaign, IL: Wolfram Media.
55. Weisstein, E. W. (2002). *Elementary cellular automaton*. Retrieved from http://mathworld.wolfram.com/ElementaryCellularAutomaton.html.
56. Wolfram, S. (1999). *The mathematica book* (4th ed.). Cambridge, UK: Cambridge University.
57. Wolfram, S. (February 2010). *Computing a theory of all knowledge*. Retrieved from https://www.ted.com/talks/stephen_wolfram_computing_a_theory_of_everything.